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Fatigue-crack propagation in vinyl urethane polymers

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The fatigue properties of a series of vinyl urethane polymers have been examined by determining the crack propagation rates as a function of the applied stress intensity factor. Measurements were also made of the craze lengths at the tips of the fatigue cracks, and the craze stress and crack opening displacement calculated. A model for fatigue-crack propagation based on cumulative damage within the crazed material has been proposed which is consistent with the observed results.

1. Introduction

One of the most common forms of failure of materials in practical use is by fatigue. Failure of this nature occurs by the action of stresses below the yield or fracture stress which are applied to the component in question in a cyclic manner. The effect of such cyclic stresses is to initiate microscopic cracks at centres of stress concentration within the material or on the surface, and subsequently to enable these cracks to grow, resulting in eventual failure of the component. This paper is concerned with the propagation stage of this process.

Fracture mechanics has been applied successfully to fatigue-crack growth in metals [1-6] and similar principles have been applied to the fatigue of polymers [7-14]. In these investigations, the crack growth rates are expressed in terms of fracture mechanics parameters which relate the applied loading to the particular crack and specimen geometry. The detailed approach differs with different investigators. For example, Lake and Lindley [8] developed the tearing energy concept of fracture proposed by Rivlin and Thomas [15], and in a rather similar fashion, Andrews and Walker [12] expressed their results in terms of a surface work parameter. In this paper, on the other hand, the applied stress during the loading cycle is expressed in terms of the stress intensity factor at the crack tip, this being used as a criterion for crack growth. Following previous workers, the fatigue-crack growth rate is expressed in the form of an empirical relationship

$$\frac{da}{dN} = A(\triangle K)^m$$

where a = crack length, N = number of cycles, $\triangle K$ = range of the stress intensity factor, A and m = constants depending on the material and test conditions.

This equation is the most general form of the law proposed by Paris [5] for predicting fatigue-crack growth rates in metals. It has been shown to describe the results in certain polymers [10, 11, 14] although the significance of the two constants A and m, with regard to the mechanisms involved, is still in doubt.

In this paper, the increments of crack growth are related to the lengths of the primary craze present at the crack tip, and a model is proposed involving the cumulative damage of the crazed material, which allows the crack to grow by certain amounts determined by the degree of damage in the craze.

2. Theory

In order to study crack growth mechanisms, the conditions at and near the crack tip must be examined closely. High stress concentrations occur in regions close to the tips of cracks, and in metals these induce local plastic yielding, such regions being known as plastic zones. Dugdale [16] proposed a mathematical model to describe the plastic zone in metals. Recent work suggests that this can also be applied to the primary craze at the crack tip in a polymer [17]. The Dugdale model provides a method whereby the

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crack (tip) opening displacement or COD can be calculated. The COD, usually designated by δ_t, is the amount by which the faces of the crack can move apart (by means of plastic flow or crazing at the crack tip) without extension of the crack and can, therefore, be regarded as a criterion for fracture.

In common with other studies of glassy polymers (see for example [18-20]), and following the approach of a previous paper on the fracture of these vinyl urethane polymers [21], it will be assumed that linear elastic fracture mechanics theory is applicable. Viscoelastic behaviour is accounted for in very simple terms, by assuming that the modulus and craze stress (and possibly the COD) are strain-rate dependent.

We will follow the treatment given by Rice [22]. This considers the situation in an infinite plate with a crack lying along the x_1 direction loaded by a uniform stress σ applied at infinity in the x_2 direction. The components of the stress field are then given by

$$\sigma_{ij} = \frac{K_1}{(2\pi r)^{\frac{1}{2}}} f_{ij}(\theta)$$

where $\sigma_{ij} = \sigma_{11}$, σ_{22} and σ_{12}, $r = [(x_1 - r_y)^2 + x_2^2]^{\frac{1}{2}}$, r_y = plastic zone size, $f_{ij}(\theta)$ are simple trigonometric functions and K_1 is the stress intensity factor.

Yielding of the material at the crack tip is considered as making the crack longer by the length r_y of the plastic zone, and there is a series of internal stresses of magnitude σ_0 in the x_2 direction acting on the extended crack surface over the region $0 < x_1 < r_y$ (x_1 is measured from the crack tip). These internal stresses are chosen so that the stress singularities produced by the two stress fields just cancel, and no stresses exist in excess of the yield stress of the material.

The length of the plastic zone derived by Rice is then given by

$$r_y = \frac{\pi}{8}\left(\frac{K_1}{\sigma_0}\right)^2 \qquad (1)$$

and the corresponding separation distance, δ, in the plastic zone is

$$\delta = \frac{8\sigma_0 r_y}{\pi E^*}\left[\xi - \frac{x_1}{2r_y}\log_e\left(\frac{1+\xi}{1-\xi}\right)\right]$$

where

$\xi = [1 - (x_1/r_y)]^{\frac{1}{2}}$,

E^* = reduced modulus,

$= E$ = Young's modulus in plane stress

$= \dfrac{E}{1-\nu^2}$ in plane strain, where ν is Poisson's ratio.

The crack opening displacement δ_t is the value of the separation distance δ at the crack tip where $x_1 = 0$, and is, therefore,

$$\delta_t = \frac{8\sigma_0 r_y}{\pi E^*} = \frac{K_1^2}{\sigma_0 E^*} \qquad (2)$$

from Equation 1.

When using this model to describe the craze at the crack tip, we assume that σ_0 takes the value of the craze stress, σ_c. Thus the craze length r_c is given by

$$r_c = \frac{\pi}{8}\left(\frac{K_1}{\sigma_c}\right)^2 \qquad (3)$$

where $K_1^2 = E^*\sigma_c\delta_t$.

3. Experimental

3.1. Materials

The fatigue measurements were carried out on a series of vinyl urethane polymers with a NCO:OH ratio of 4:3 at styrene concentrations of 20, 50 and 70%. This work formed part of a larger programme by which it was hoped to establish the effects of the pre-polymer chain length (determined by the NCO:OH ratio) and the styrene concentration on the mechanical properties [23].

3.2. Apparatus

Most of the fatigue experiments were undertaken using a pneumatic uniaxial tensile fatigue tester, which was designed and built by Imperial Chemical Industries Ltd, Plastics Division, Welwyn Garden City. The specimen is held between two clamps, one of which is fixed, and the other attached to a piston rod. By directing air at pressure to either side of the piston, a cyclic load may be applied to the specimen. The magnitude of the applied load is determined from the air pressure and the size of the piston. For these experiments, the piston size was 1 in. diameter. The load is applied in the form of a square wave, the rise time of which depends on the response of the pneumatics and which was estimated as being in the range 30 to 50 msec.

The pneumatic system was designed to apply loads varying from compression to tension in each cycle, or from zero to tension, but not

from a low tension to a high tension. The frequency could vary in the range from 5 Hz to one cycle every 36 h.

A few tests were also performed on a Servo Consultants tensile testing machine made available by ICI Corporate Laboratory, Runcorn. This machine was hydraulically operated and capable of great flexibility both in terms of frequency and cyclic waveform. In the present work the load was applied as a saw-tooth waveform at a frequency of 1 Hz.

All the fatigue tests were performed at room temperature.

3.3. Specimen geometry

Parallel and tapered cleavage specimens were used for the crack propagation rate measurements (Figs. 1 and 2). It was found necessary to introduce side grooves in order to constrain the crack to a flat trajectory across the specimen. These grooves were approximately 1 mm deep and 0.006 in. wide. To assist in starting the crack a "swallow-tail" crack front was produced by cutting right through the specimen at the ends (Fig. 3).

The introduction of side grooves disturbs the stress field at the crack tip, so that some form of correction is necessary when calculating the value of the stress intensity factor at the crack tip. This was done following the method suggested by Marshall *et al* [19], which involved performing a slow cleavage test in tension on a specimen, and measuring the load and deflection in the specimen for different crack lengths. From such measurements, the rate of change of compliance with crack length was calculated, and the value of the stress intensity factor calculated for various crack lengths using the Irwin-Kies relation [24]:

$$K^2 = \frac{P^2 E^*}{2B_c} \frac{dc}{da}$$

where P = tensile load, B_c = crack width, dc/da = rate of change of compliance with crack length. By comparing the value of the stress intensity factor calculated in this way, with those values calculated from solutions obtained from boundary collocation procedures [25, 26], it was found that the introduction of side grooves reduces the stress intensity factor by ~ 0.63.

Some small parallel cleavage specimens were used for the craze length measurements. These measured 3 in. × $2\frac{3}{4}$ in. × $\frac{1}{8}$ in., and were also grooved.

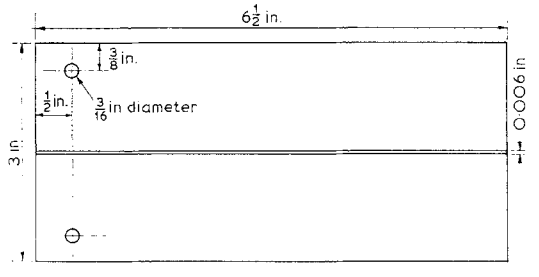

Figure 1 Parallel cleavage specimen geometry.

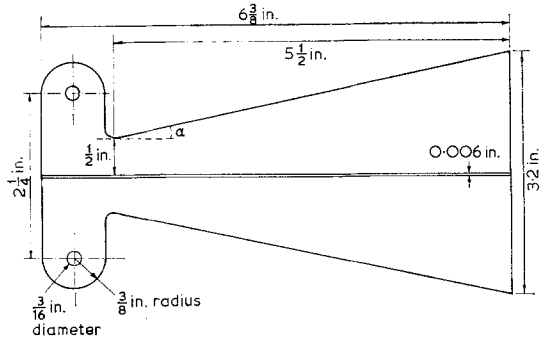

Figure 2 Tapered cleavage specimen geometry.

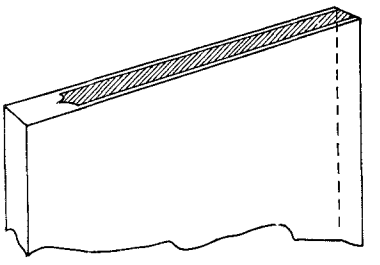

Figure 3 Schematic diagram of a fatigue fracture surface showing the "swallow-tail" crack front.

3.4. Measurements of crack propagation and craze size

The crack propagation rate was measured by noting the position of the crack front at regular intervals, by means of a travelling microscope. The microscope carriage was fitted with a micrometer head enabling the crack length to be measured to within 0.01 mm.

For the craze length measurements, small parallel cleavage specimens were tested in the fatigue tester, and the test stopped when the crack growth rate had reached the desired value.

The specimen was then removed from the tester, and a small section surrounding the crack tip was carefully cut out (dotted region in Fig. 4), polished on a buffing wheel and examined under an optical microscope. The crack tip was examined by looking vertically down onto the crack surface in a direction normal to the crack plane (arrow in Fig. 4), using both transmitted and reflected illumination.

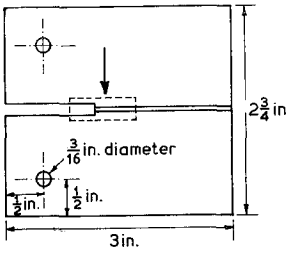

Figure 4 Small parallel cleavage specimen geometry showing the region surrounding the crack tip (dotted) which is used for examination of the craze.

4. Results and discussion

4.1. Crack propagation characteristics

From measurements of the crack length and the number of cycles elapsed, the crack growth rate was calculated and plotted as a function of the stress intensity factor, $\triangle K$. $\triangle K$ was calculated using the boundary collocation procedures of Gross and Srawley [25, 26], after applying the correction described above to take account of the grooving. The value of the load used in the calculations was the maximum tensile load. The crack growth characteristics for the three resins are shown in Fig. 5.

There appear to be two regions where the crack growth rate increases with increasing $\triangle K$, separated by a transition region where the crack growth rate increases by two orders of magnitude for very little increase in $\triangle K$. The gradual increase from the plateau (transition) region is similar to that observed in the tensile fracture of polymethylmethacrylate [19, 27] in which the crack velocity was measured as a function of the fracture toughness.

The fatigue surfaces showed clear striation markings, which indicated the successive positions of the crack front. In order to see if the crack was advancing each cycle, the striation spacing was compared with the measured crack growth rate. Excellent agreement was found for crack growth rates greater than 10^{-4} mm cycle^{-1} indicating that in that range, the crack was advancing each cycle. To test this point at lower growth rates, it was necessary to prepare gelatine replicas of the fatigue fracture surfaces for examination in the electron microscope. These replicas showed that for growth rates less than 10^{-5} mm cycle $^{-1}$, the striation spacing was considerably greater than the measured crack growth rate, by up to two orders of magnitude. This discrepancy between the striation spacing and the crack growth rate has also been ob-

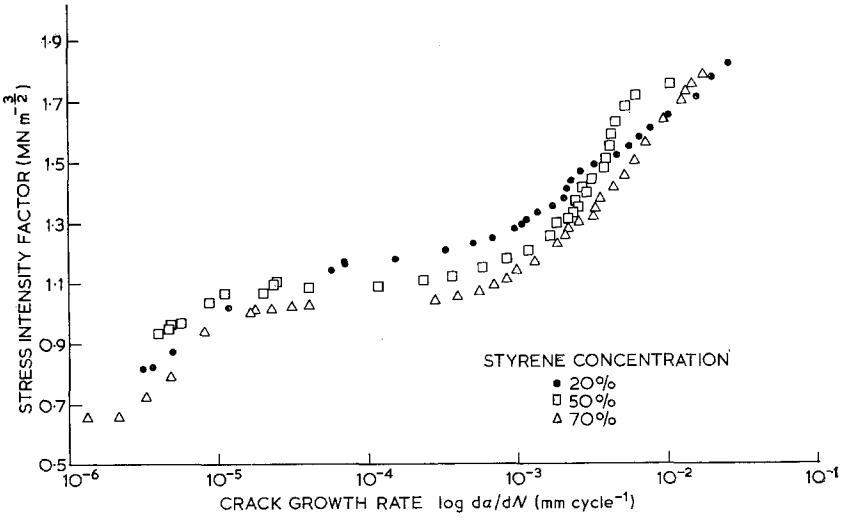

Figure 5 Fatigue-crack growth characteristics in the vinyl urethane polymers.

served in metals [4], where it was proposed that crack growth was occurring inhomogeneously, only small sections of the crack front advancing during any one cycle. In this way several cycles would be required before the entire crack front had advanced one striation. Although this could be the explanation for the present observations, it is also possible to postulate another mechanism by which the crack configuration changes so much in a polymer when the crack grows, that several cycles are required to condition the material so that further growth may take place. Thus, when crack growth does take place, the crack advances by one striation at a time; however, such growth does not occur on every cycle.

In terms of the plots in Fig. 5, the plateau region marks the transition from this slow growth behaviour to the faster growth behaviour where a 1:1 correlation exists between the striation spacing and the growth rate.

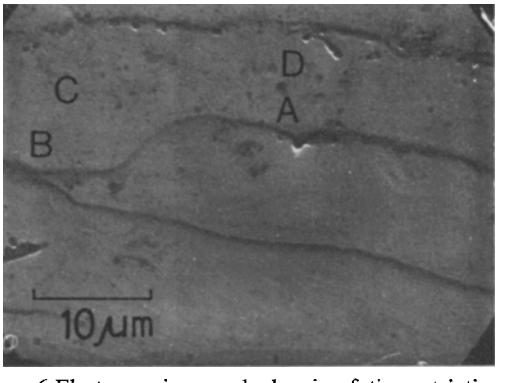

Figure 6 Electron micrograph showing fatigue striations in a vinyl urethane polymer. Crack growth rate is 10^{-2} mm cycle^{-1}.

A further interesting feature was observed during the electron microscope studies, and is shown in Fig. 6. This micrograph was taken from a crack travelling at the rate of 10^{-2} mm cycle^{-1}. It can be seen that one striation bends back onto the previous one. It appears that part of the crack (A) advanced during one cycle while another part (B) was held back for some reason. This could be caused by small inhomogeneities within the material. On the next cycle the whole crack front advanced as one, so that the measured spacing in one region (C) was almost twice that in the other region (D). It, therefore, appears that inhomogeneous growth does occur, although it should be mentioned that no such effect was observed on any of the slow growth specimen surfaces.

It is clear that the Paris equation [5]

$$\frac{da}{dN} = A(\triangle K)^m \tag{4}$$

may only be true over a limited range of the results shown in Fig. 5. The present results were obtained over a much wider range of crack growth rates than hitherto, and even when the results are replotted using logarithmic scales for both ordinate and abscissa, it is clear that a power law of this nature only holds to a first approximation at high crack speeds. This corresponds to the range in which there is a 1:1 correlation between the striation spacing and the crack growth rate. Fig. 7 shows the results for the polymer with 70% styrene concentration, and a straight line has been fitted to the results for crack growth rates greater than 10^{-3} mm cycle^{-1}. Similar graphs were plotted for the other resins, and the slopes of the straight lines gave the values of m of 5.5, 4.0 and 9.5 for polymers with styrene concentrations of 70, 50 and 20%, respectively. Experiments on metals have shown that m varies between 2 and 7 except in a few cases where higher values were measured [28], the most common value being 4. Similar values have been observed in polymers [8, 12, 29] and have been reviewed by Andrews [30]. The value of 9.5 for the polymer with 20% styrene concentration seems extraordinarily high. Moreover, the curve for this resin shown in Fig. 5 crosses over the curves for the other two resins at high crack growth rates, as the crack exhibits an unusually high rate of acceleration. We believe that this anomalous behaviour may be attributed to the large number of voids present in this polymer. During the preparation, the uncured polymer has a very high viscosity owing to the long pre-polymer chain length and the low styrene concentration, so that air bubbles are unable to escape very easily even under vacuum. On curing, these bubbles are frozen into the polymer and can affect the crack growth characteristics.

For this reason, detailed investigations of the crack growth mechanisms have been concentrated on the polymers with styrene concentrations of 50 and 70%.

4.2. Effect of frequency on crack growth rate

Preliminary tests on simple dumb-bell shaped specimens showed that the striation spacing was

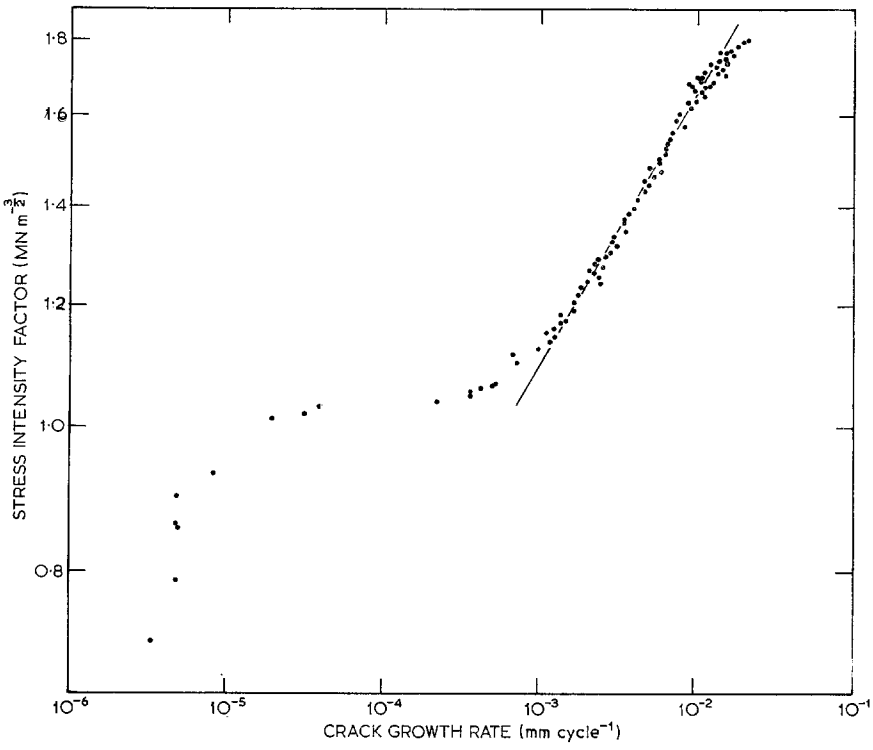

Figure 7 Fatigue-crack growth characteristics in a high styrene (70%) vinyl urethane polymer showing the application of the Paris Equation 4, see [5].

identical for tests carried out at frequencies of 1 and 5 Hz. This result was confirmed by tests using tapered cleavage specimens in which frequencies in the range from 5 Hz down to one cycle every 2 min were investigated, and no discernible effect was apparent. Tapered specimens were used since their geometry is such that the stress intensity factor at constant load, is almost independent of crack length over a large range of crack lengths. From this result it was concluded that crack growth occurs during a relatively brief portion of the test cycle and that growth is terminated owing to changes in conditions at the crack tip rather than owing to removal of the applied external load. This means that fatigue crack growth is cycle dependent rather than time dependent. This result is similar to that observed in polymethylmethacrylate [31] where it was found that the crack would stop while the full load was still maintained and would not continue to grow until the load had been removed and then re-applied. It must be emphasized that changes in frequency using the testing techniques adopted here do not imply changes in *strain-rate*, only in the time for which the load is applied. The strain-rate does affect the growth rate as will be shown below. Using the pneumatic equipment described above, the strain-rate is determined by the response of the pneumatics and, therefore, may be affected by changes in the air pressure in the piston, being higher at higher pressures.

The independence of the crack growth rate on the frequency also confirms that there are no significant heating effects. Polymers are sensitive to both strain-rate and temperature, and at sufficiently high testing frequencies it would be expected that heat will be generated faster than it can be conducted away resulting in a local rise in temperature [32-34]. The results show that even at 5 Hz there are no detectable heating effects.

4.3. Craze length measurements

Fig. 8 shows a micrograph taken in transmission where the crack tip was observed on a section cut out from a small parallel cleavage specimen in the manner described in Section 3.4 above. The fatigue striations formed immediately prior

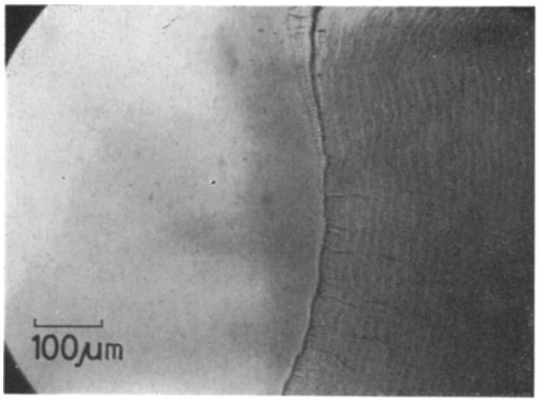

Figure 8 Transmission optical micrograph of a stationary fatigue crack showing fatigue striations.

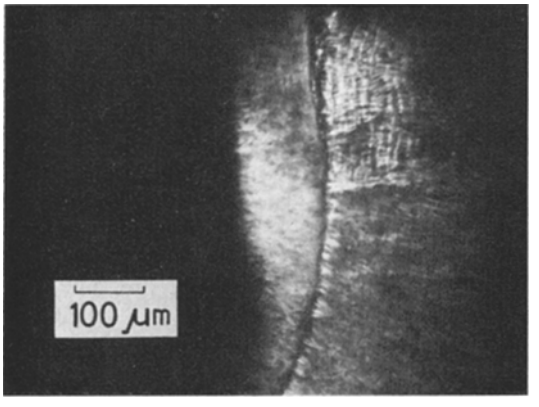

Figure 9 Reflection optical micrograph of a stationary fatigue crack (the same as in Fig. 8), showing the craze at the crack tip.

to the termination of the fatigue tests are clearly visible, but there is no sign of any craze ahead of the crack. Fig. 9 shows a similar micrograph taken using reflected illumination, where the craze is now clearly shown up ahead of the crack tip, light being reflected from the interface between the crazed and uncrazed material. From micrographs such as this, the craze length can easily be measured. Fig. 10 shows the same specimen using a lower power objective and it can be seen that crazing is most intense at the outer edges of the crack front, decreasing in intensity towards the middle. This is a direct result of introducing side-grooves to constrain the crack which introduces additional stress concentrations. The profile of the crack front shows it to be leading at the edges, again a result of the side-groove stress concentrations. The

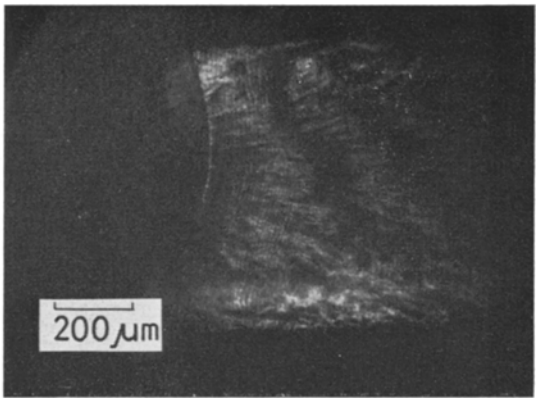

Figure 10 Reflection optical micrograph of a stationary fatigue crack (the same as in Figs. 8 and 9), showing the crack and craze profile.

striation spacing, which for the specimen shown gives a direct measure of the crack growth rate, is considerably less than the craze length. In this case, the growth rate was approximately 20 μm per cycle, while the craze length is approximately 130 μm.

In a previous paper, the fracture behaviour of these polymers is reported, and the results analysed in terms of the structure of the craze at the crack tip using the Dugdale model for a plastic zone [21].

On this model, the fracture toughness (i.e. the critical value of the stress intensity factor when the crack starts to grow) is determined by three basic material parameters, the crack opening displacement δ_t, the craze stress σ_c and the modulus E^*. The stress intensity factor K_1 is given by

$$K_1^2 = E^* \sigma_c \delta_t \tag{5}$$

and the craze length

$$r_c = \frac{\pi}{8}\left(\frac{K_1}{\sigma_c}\right)^2 . \tag{3}$$

We will now attempt to explain the fatigue behaviour in similar terms. In the case of fatigue, a loading programme is applied which gives rise to a stress field described by a stress intensity factor K, which is less than K_1, the critical stress intensity factor for crack propagation in a fracture test. Both the craze stress σ_c and the modulus E^* will depend on temperature and strain-rate in the loading cycle. We can then regard the crack opening displacement δ_t and the craze length r_c as dependent variables,

depending on the craze stress and the stress intensity factor for the particular loading conditions employed.

On this view the two equations above can be used to calculate the craze stress and the crack opening displacement from the known values of the stress intensity factor in the fatigue experiment (calculated from the applied load and the geometry of the specimen) and the craze length r_c (determined experimentally). The results for the polymers with 50 and 70% styrene concentrations are shown in Table I. It can be seen that the predicted craze stress is in both cases very close to a constant value over the whole range of stress intensity factor. The values do decrease somewhat with increasing stress intensity factor, which we believe to be owing to slight increases in strain-rate brought about by the slightly higher loads which were necessary. It is to be noted that the craze stress is greater for the 50% styrene concentration polymer, a result which is consistent with the previous work on the tensile fracture of these polymers [21] where it was found that the craze stress decreased monotonically with increasing styrene concentration. This difference in craze stress manifests itself in Table I in differences in craze length and crack opening displacement and, in Fig. 5 in differences in crack growth rates.

The crack opening displacement and craze length both decrease markedly with increasing stress intensity factor. The rapid increase in the crack growth rate with increasing stress intensity factor must be influenced by these parameters and the explanation for this behaviour must be sought in the change in these parameters. Comparing Fig. 5 with Table I shows that with the exception of very fast cracks, the increments of crack growth are very much smaller than the craze lengths. In the case of very fast fatigue cracks, the test approaches a tensile fracture test. This shows that craze fracture only occurs as a result of cumulative damage in material within the craze which has been subjected to many cycles of loading and unloading. Under these conditions, it is not surprising that the crack growth rate increases rapidly at high crack opening displacements. The material in the craze is similar in many respects to the uncrazed material (except that it possesses a lower density due to voiding). It is, therefore, expected that at very low cyclic strains, the crazed material behaves viscoelastically with a high degree of recovery. At higher strains, however, molecular orientation occurs with permanent plastic flow, these effects increasing rapidly with increasing strain as is observed in conventional tensile testing. The structure of the material within the craze, will, therefore, be expected to vary depending on the size of the imposed cyclic strains. Near the crack tip, the strains are highest, and the crazed material suffers most damage, the precise way in which this damage varies along the craze length being determined by the shape of the craze, i.e. the value of the ratio δ_t/r_c. The size of the growth increments will depend on the amount of craze which has been sufficiently badly damaged.

The implication is that the crack growth depends on the crack moving forward through the craze from a point where the separation is δ_1 at the beginning of the loading cycle to a point

TABLE I

Material	$\triangle K$ (MN m$^{-3/2}$)	r_c (μm)	σ_c (MN m^{-2})	δ_t (μm)
50% styrene	1.8	87.44	121	8.49
	1.24	45.63	115	4.25
	1.09	40.94	107	3.53
	1.03	34.56	110	3.06
	0.85	35.91	89	2.58
	0.80	37.30	82	2.48
70% styrene	1.72	137.58	92	10.20
	1.67	131.85	91	9.72
	1.65	130.81	91	9.49
	1.20	100.00	75	6.09
	0.90	73.56	66	3.89
	0.77	55.52	65	2.88
	0.70	62.11	56	2.77
	0.60	23.77	77	1.48

where it is δ_2, i.e. material in the range δ_1 to δ_2 fails during each cycle. Comparing the two polymers, the growth rate at any given value of the stress intensity factor is determined by the ratio δ_1/r_c, the growth rate being higher, the smaller this ratio.

From Equations 2 and 3

$$\frac{\sigma_t}{r_c} = \frac{8}{\pi}\frac{\sigma_c}{E^*} \cdot \quad (6)$$

It is, therefore, expected that the polymer containing 50% styrene concentration would show slower growth rates since that polymer has the higher craze stress. As Fig. 5 shows, this is in fact observed.

4.4. Effect of strain-rate on fatigue behaviour

It was shown in the previous section that the craze stress is a crucial parameter in determining the crack growth rate since, under given loading conditions, the craze stress determines the length of craze ahead of the crack. The effect of strain-rate on the craze stress was mentioned briefly, and we now wish to report some additional experiments which were performed in an attempt to clarify this effect. These experiments were performed using a Servo Consultants Hydraulic Tensile Tester. The loading programme chosen was to apply the load in the form of a saw-tooth waveform at a frequency of 1 Hz, so that the rise time of the applied load was approximately 0.5 sec, compared with 30 to 50 msec using the pneumatic uniaxial tester. These experiments, therefore, provided a reduction in the effective applied strain-rate of about one order of magnitude. Parallel cleavage specimens were used and attempts made to measure the crack growth rates as a function of the applied stress intensity factor as before. Although zero drift in the load cell calibration made this rather difficult, reliable measurements were made of the growth rates at a stress intensity factor of 1.25 $MNm^{-3/2}$. Table II shows the appropriate values for the polymer containing 50% styrene concentration, together with the equivalent figures obtained using a square waveform, from Table I. It will be noted that the growth rate has increased by almost one order of magnitude by using a slower strain-rate, and that there is a dramatic fall in the calculated craze stress from 115 to 70 MNm^{-2}. The craze length and crack opening displacement are both correspondingly higher. These results are in accordance with what we would expect on the basis of the model outlined in Section 4.3.

4.5. Evidence for "memory" of the craze

The arguments presented above for craze formation and the subsequent fracture of the craze during fatigue testing, imply that the structure of the material within the craze varies considerably along the length of the craze. Those parts of the craze furthest from the crack tip suffer very little owing to the relatively low cyclic strains there; however, nearer the crack tip, as the separation of the "craze/no craze" interfaces increases, higher cyclic strains operate, resulting in permanent damage to the craze structure which accumulates in the manner described above. The variation in structural damage is unique to a given set of loading conditions and it will, therefore, be expected that the craze will retain some "memory" of previous loading conditions.

This effect was demonstrated by comparing fatigue with the extreme condition of cleavage fracture. A parallel cleavage specimen was pulled on an Instron Tensile Tester to produce a crack. The specimen was then transferred to the pneumatic fatigue tester and the crack allowed to proceed under fatigue conditions. The fracture surface observed for this combined test is shown in Fig. 11. The spacing of the striations to the left of the boundary "A" gradually decreases over a distance of approximately 50 μm, before settling down to a more or less constant value which gradually increases again towards the extreme left of the micrograph. On a similar specimen the craze length at the tip of a similar Instron crack was measured as being approximately 50 μm. Therefore, it appears that the

TABLE II

Waveform	$\triangle K$ ($MN\ m^{-3/2}$)	Growth rate (mm $cycle^{-1}$)	r_c (μm)	σ_c ($MN\ m^{-2}$)	δ_t (μm)
Saw-tooth	1.25	1.0×10^{-2}	120.00	71	6.98
Square	1.24	1.7×10^{-3}	45.63	115	4.25

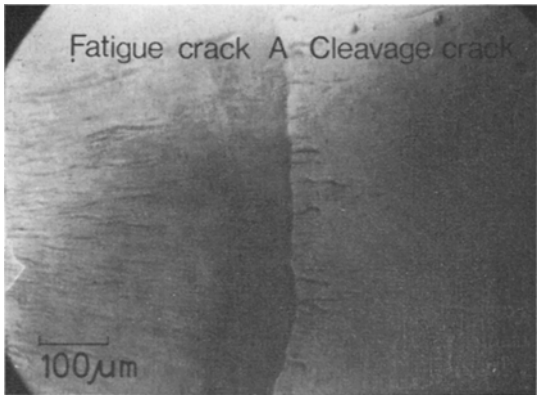

Figure 11 Transmission optical micrograph showing the growth of a fatigue crack from a cleavage crack.

fatigue crack had to cut right through the initial crazed material before the memory of the Instron test was completely removed. This experiment confirms the importance of the memory of the craze and emphasizes that particular loading conditions produce a particular craze structure which then influences the subsequent crack growth characteristics. This memory is contained in the entire length of the craze.

5. Final discussion and conclusions

The experiments described above emphasize the importance of the structure of the primary craze in determining the fatigue characteristics of polymers. The important parameters are the reduced modulus E^*, the craze stress σ_c, and the loading conditions described by the stress intensity factor $\triangle K$. From these parameters the crack opening displacement δ_t may be derived. The length of craze ahead of the crack is determined by the craze stress. Under cyclic loading conditions, the craze suffers from cumulative damage, the extent of which is very dependent upon the size of δ_t and the craze length. From Equation 2,

$$\delta_t \propto K^2$$

so that a model of this sort predicts that the growth rate, da/dN, varies as δ_t^2 if the constant m in the Paris equation (Equation 4) takes the most common value of 4.

Under given loading conditions, a crack opening displacement is imposed on the system, and this combined with the craze length determines the form of the craze damage which accumulates during subsequent loading cycles. Eventually the damage in the craze immediately adjacent to the crack tip becomes sufficient for the crack to grow into the craze. It is reasonable to assume that craze growth occurs at a much slower rate than crack growth, so that for a brief period of time, the craze length is reduced. The crack eventually stops on encountering craze material which has not yet reached breaking point owing to insufficient accumulation of damage. During the remainder of that half of the cycle, fresh craze grows ahead of the old craze. During the second half cycle when the load is removed, the system relaxes before the process is repeated on the next cycle.

Frequency is only an important factor when (a) heating effects are induced, or (b) when the strain-rate is affected. The main effect of strain-rate is to vary the craze stress, although we might also expect some changes in the reduced modulus, and to a greater extent, the rate of accumulation of damage within the craze. The effect of the waveform shape can also be primarily attributed to changes in strain-rate.

It was noted that, at low values of the stress intensity factor, there was a significant discrepancy between the striation spacing and the measured crack growth rate. In view of the arguments presented above, it seems quite possible for the crack to require several cycles, even hundreds of cycles, before sufficient damage has accumulated to allow further growth of the crack.

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